NAG Toolbox for MATLAB

g05qa

1 Purpose

g05qa generates a random orthogonal matrix.

2 Syntax

```
[a, iseed, ifail] = g05qa(side, init, m, a, igen, iseed, 'n', n)
```

3 Description

g05qa pre- or post-multiplies an m by n matrix A by a random orthogonal matrix U, overwriting A. The matrix A may optionally be initialized to the identity matrix before multiplying by U, hence U is returned. U is generated using the method of Stewart 1980. The algorithm can be summarized as follows.

Let $x_1, x_2, \ldots, x_{n-1}$ follow independent multinormal distributions with zero mean and variance $I\sigma^2$ and dimensions $n, n-1, \ldots, 2$; let $H_j = \mathrm{diag}\left(I_{j-1}, H_j^*\right)$, where I_{j-1} is the identity matrix and H_j^* is the Householder transformation that reduces x_j to $r_{jj}e_1$, e_1 being the vector with first element one and the remaining elements zero and r_{jj} being a scalar, and let $D = \mathrm{diag}(\mathrm{sign}(r_{11}), \mathrm{sign}(r_{22}), \ldots, \mathrm{sign}(r_{nn}))$. Then the product $U = DH_1H_2 \ldots H_{n-1}$ is a random orthogonal matrix distributed according to the Haar measure over the set of orthogonal matrices of n. See Theorem 3.3 in Stewart 1980.

One of the initialization functions g05kb (for a repeatable sequence if computed sequentially) or g05kc (for a non-repeatable sequence) must be called prior to the first call to g05qa.

4 References

Stewart G W 1980 The efficient generation of random orthogonal matrices with an application to condition estimates *SIAM J. Numer. Anal.* **17** 403–409

5 Parameters

5.1 Compulsory Input Parameters

1: **side – string**

Indicates whether the matrix A is multiplied on the left or right by the random orthogonal matrix U.

side = 'L'

The matrix A is multiplied on the left, i.e., pre-multiplied.

side = 'R'

The matrix A is multiplied on the right, i.e., post-multiplied.

Constraint: **side** = 'L' or 'R'.

2: **init – string**

Indicates whether or not **a** should be initialized to the identity matrix.

init = 'I'

a is initialized to the identity matrix.

init = 'N'

a is not initialized and the matrix A must be supplied in **a**.

Constraint: **init** = 'I' or 'N'.

3: **m – int32 scalar**

m, the number of rows of the matrix A.

Constraints:

if **side** = 'L', **m** > 1;
m ≥ 1 otherwise.

4: **a(lda,n) – double array**

lda, the first dimension of the array, must be at least **m**.

If **init** = 'N', **a** must contain the matrix A.

5: **igen – int32 scalar**

Must contain the identification number for the generator to be used to return a pseudo-random number and should remain unchanged following initialization by a prior call to g05kb or g05kc.

6: **iseed(4) – int32 array**

Contains values which define the current state of the selected generator.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **a**.

n, the number of columns of the matrix A.

Constraints:

if **side** = 'R', **n** > 1;
n ≥ 1 otherwise.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, wk

5.4 Output Parameters

1: **a(lda,n) – double array**

The matrix UA when **side** = 'L' or the matrix AU when **side** = 'R'.

2: **iseed(4) – int32 array**

Contains updated values defining the new state of the selected generator.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, **m** < 1,
or **n** < 1,
or **lda** < **m**.

ifail = 2

On entry, **side** ≠ 'L' or 'R',
or **init** ≠ 'I' or 'N'.

ifail = 3

On entry, an orthogonal matrix of dimension 1 has been requested.

7 Accuracy

The maximum error in $U^{\mathrm{T}}U$ should be a modest multiple of ***machine precision*** (see Chapter X02).

8 Further Comments

g05qb computes a random correlation matrix from a random orthogonal matrix.

9 Example

```
side = 'Right';
init = 'Initialize';
m = int32(4);
a = zeros(4, 4);
igen = int32(1);
iseed = [int32(1762543);
     int32(9324783);
     int32(423446);
     int32(742355)];
[igen, iseed] = g05kb(igen, iseed);
[aOut, iseedOut, ifail] = g05qa(side, init, m, a, igen, iseed)
```

```
aOut =
   -0.2191   -0.1969   -0.4131   -0.8618
    0.4379    0.0055    0.7616   -0.4776
    0.6993   -0.6266   -0.3223    0.1199
   -0.5208   -0.7540    0.3813    0.1219
iseedOut =
    12622195
     6767553
    16277754
    11529009
ifail =
           0
```